
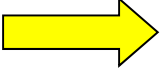


Light Source I

Takashi TANAKA (RIKEN SPring-8 Center)

CONTENTS

- Light Source I 
- Introduction
 - Fundamentals of Light and SR
 - Overview of SR Light Source
 - Characteristics of SR (1)
- Light Source II 
- Characteristics of SR (2)
 - Practical Knowledge on SR

Introduction

SR Facility and Light Source

- SR: Definition
 - Electromagnetic wave emitted by a charged particle deflected by a magnetic force
- SR Facility
 - Accelerators to generate a high-energy electron beam
 - **Magnetic devices (SR light source) to generate intense SR**
 - Optical elements (monochromators, mirrors,..)
 - Experimental stations

SR as a Probe for Research

- SR has a lot of advantages over other conventional light sources
 - Highly collimated (laser-like)
 - Wavelength tunability
 - Polarization
 -
- However, the total radiation power does not differ significantly.



Comprehensive understanding of SR (and light source) is required for efficient experiments.

Topics in This Lecture (1)

- Fundamentals of Light and SR
 - Why we need SR?
 - Physical quantity of light
 - Uncertainty of light: Fourier and diffraction limits
 - SR: Light from a moving electron
- Overview of SR Light Source
 - Types of light sources
 - Magnet configuration
- Characteristics of SR (1)
 - Radiation from bending magnets

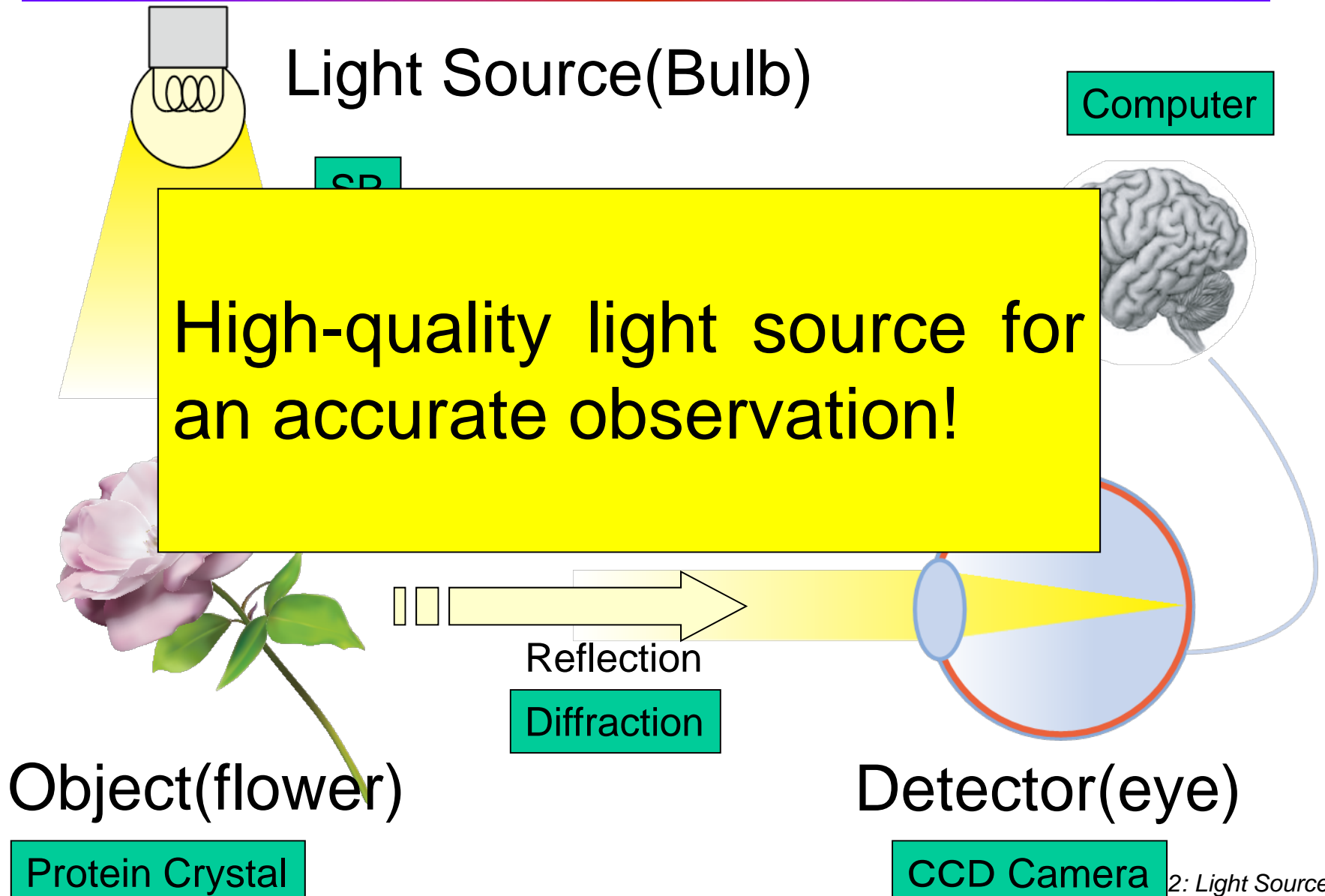
Topics in This Lecture (2)

- Characteristics of SR (2)
 - Electron Trajectory in IDs
 - Radiation from wigglers
 - Radiation from undulators
- Practical Knowledge on SR
 - Finite emittance and energy spread
 - Heat load and photon flux
 - Evaluation of optical properties of SR
 - Definition of undulators and wigglers
 - Numerical examples

Fundamentals of Light and SR

- Why we need SR?
- Physical Quantity of Light
- Uncertainty of Light
- SR: Light from a Moving Electron

Observation with Light



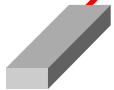
Which Quality is Better?

Specs of SPring-8

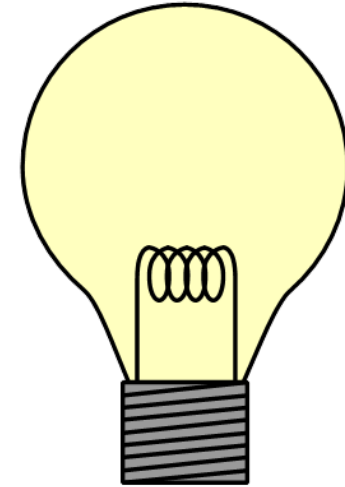
- $E = 8\text{GeV}$

- $I = 100\text{mA}$

- $L = 1500\text{m}$



1mW Laser (pointer)



100W Bulb

Lighting equipment in a room:

Bulb

Pointer during a presentation:

Laser



Depends on the Object!

How to Define the Quality of Light?(1)

- The performance of the light source depends on the dimension of the object and the method to detect light.
- For observation, the photons emitted by the light source should be
 - illuminated on the object for interaction
 - recognized by the detector for analysis

Quality of Light Source:
How efficiently the above
conditions are satisfied?

How to Define the Quality of Light?(2)

Important Features of the Light Source

	Object		Related Items
	Flower	Protein	
Radiation Power	◎	○	# Emitted Photons
Source Size	×	◎	Illuminated Area
Directivity	△	◎	
Monochromaticity	△	◎	Accuracy of Analysis



Brilliance

What is Brilliance?

$$\sim \frac{\text{Brilliance}(\text{photons/sec/mm}^2/\text{mrad}^2/0.1\% \text{B.W.})}{\text{Total Power} \times \text{Source Size} \times \text{Angular Divergence} \times \text{Band Width}}$$

- Brilliance specifies the quality of light for observation of microscopic objects.
- The brilliance of a light source with a high total power is not necessarily high.

Examples of Brilliance

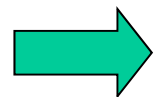
	Bulb	Laser Pointer
Total Power (W)	100	10^{-3}
Angular Div. (mrad ²)	$4\pi \times 10^6$	1
Source Size: (mm ²)	10^2	1
Bandwidth: (%)	100	0.01
Brilliance (photons/sec/....)	$\sim 10^8$	$\sim 10^{16}$

Laser is the best light source to observe the microscopic object!

X ray as a Probe

- Definition (not unique)
 - Electromagnetic wave (= light) with λ of 10 nm (10^{-8} m) \sim 0.1 Å (10^{-11} m)
- Properties
 - High Energy/Photon
 - High Penetration (Roentgen etc..)
- Application to Microscopic Objects
 - X-ray Diffraction
 - Fluorescent X-ray Analysis

• No Practical Lasers!!



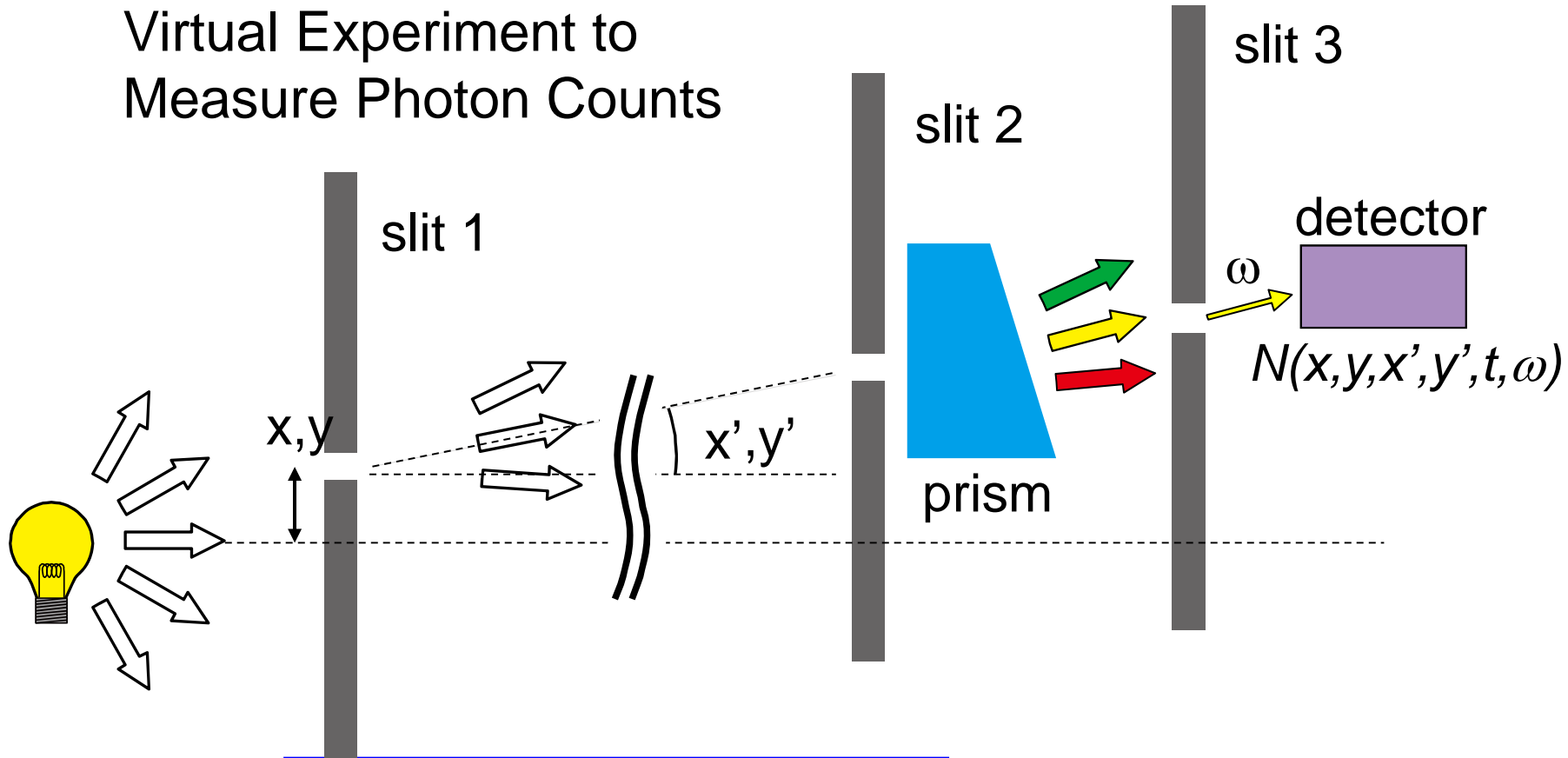
Synchrotron Radiation(SR)

Fundamentals of Light and SR

- Why we need SR?
- Physical Quantity of Light
- Uncertainty of Light
- SR: Light from a Moving Electron

Phase Space

Virtual Experiment to
Measure Photon Counts



$$n = \frac{N(x, y, x', y', \omega, t)}{\Delta x \Delta y \Delta x' \Delta y' \Delta t \Delta \omega / \omega} \equiv \frac{N}{\Delta \Omega}$$

Av. Photon Density

Volume in 6-D
Phase Space

Brilliance (Brightness)

- Brilliance (photons/sec/mm²/mrad²/0.1%B.W.) is defined as the **photon density in the 6D phase space**, i.e.,

$$B = \lim_{\Delta\Omega \rightarrow 0} n = \frac{d^6 N(x, y, x', y', t, \omega)}{dx dy dx' dy' dt d\omega / \omega}$$

- In practice, $\Delta\Omega$ can never be 0 due to uncertainty of light, thus **brilliance is not a physical quantity that can be actually measured.**

Photon Flux and Flux Density

- Removing the 1st slit gives the angular flux density (photons/sec/mrad²/0.1%B.W), i.e.,

$$\frac{d^2 F}{dx' dy'} = \iint B dx dy$$

- Removing the 1st & 2nd slits gives the total flux (photons/sec/0.1%B.W), i.e.,

$$F = \iiint B dx dy dx' dy'$$

- Estimation of number of photons to be delivered to the sample.

Radiation Power and Power Density

- Removing the 1st & 3rd slits gives the angular power density (W/mrad²), i.e.,

$$\frac{d^2 P}{dx' dy'} = 10^3 Q_e \hbar \int \frac{d^2 F}{dx' dy'} d\omega$$

↘ conversion from photons/sec/0.1%B.W. to W

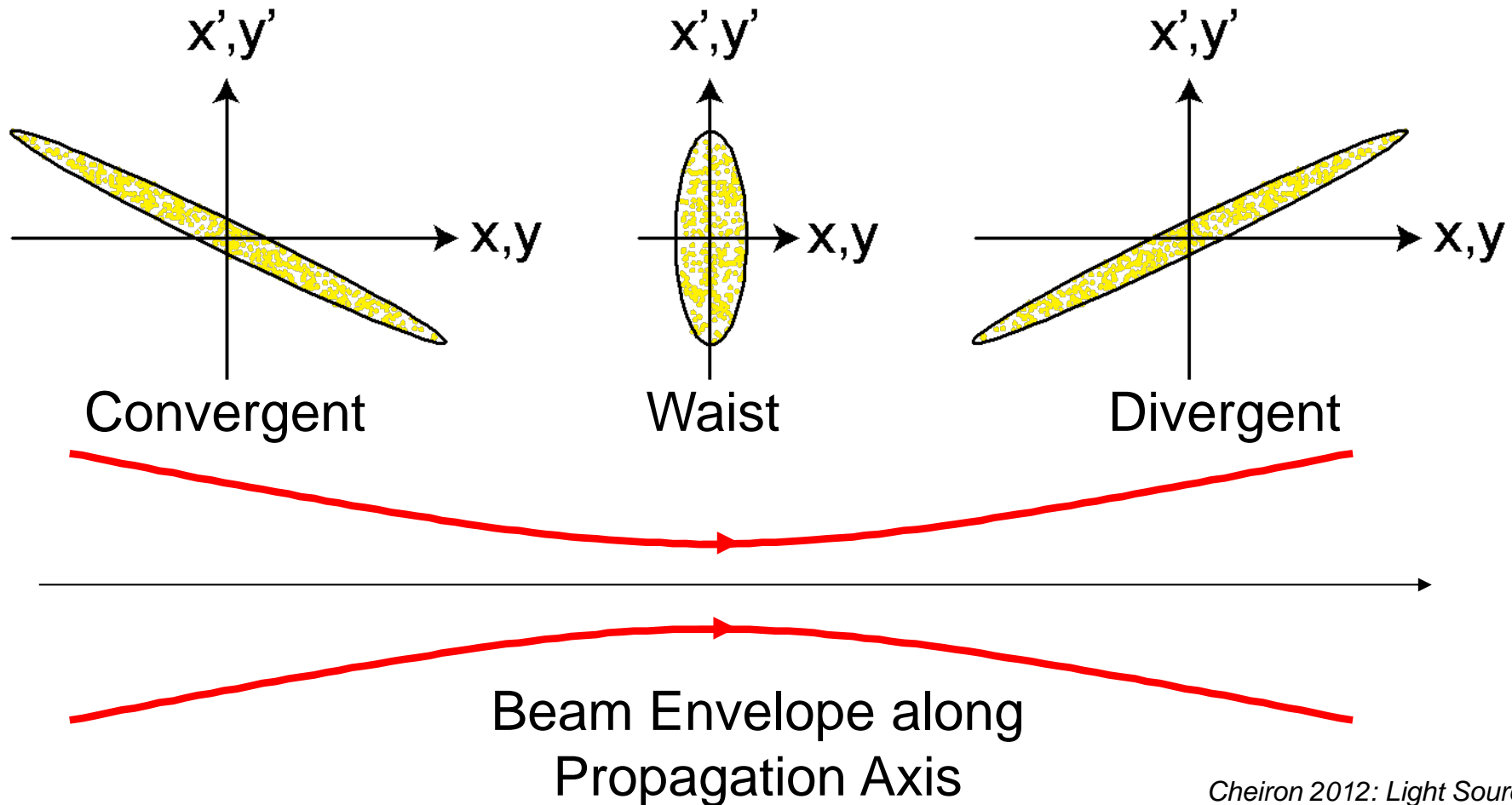
- Removing all the slits gives the total power (W), i.e.,

$$P = 10^3 Q_e \hbar \iiint \frac{d^2 F}{dx' dy'} d\omega dx' dy'$$

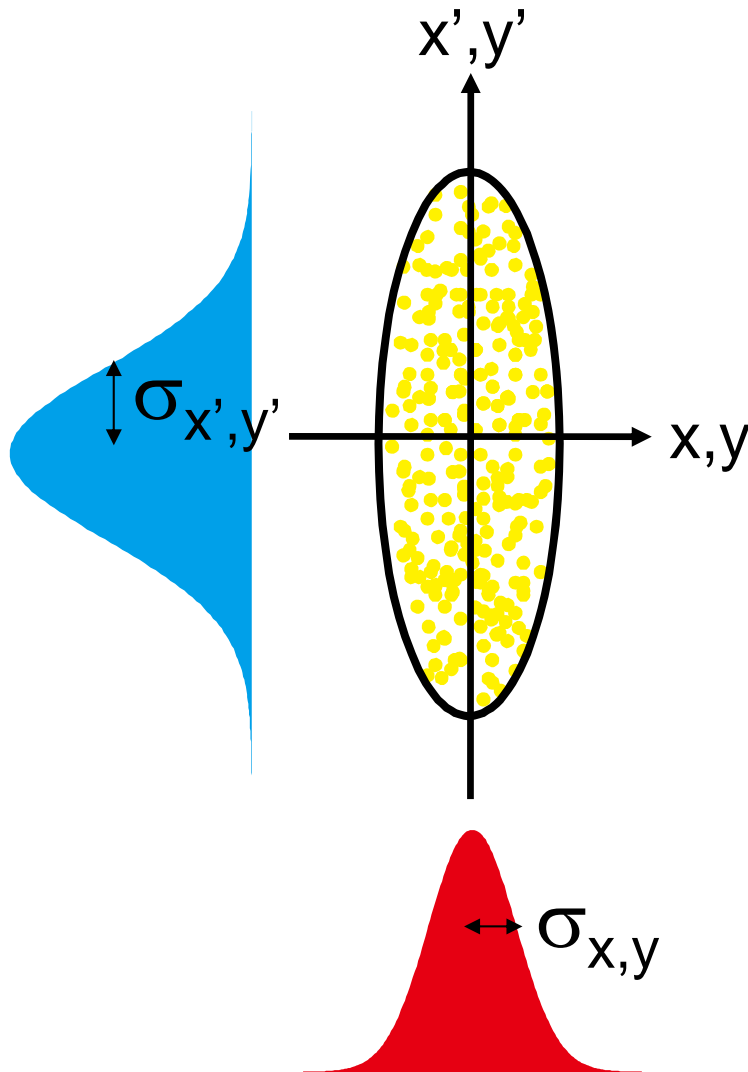
- Estimation of heat load on BL components.

Photons in 4D Phase Space

- Photon distribution in the 4-D phase space at different longitudinal positions.



Source Size, Divergence, Emittance



- Source size ($\sigma_{x, y}$) is defined as the beam envelope at the beam waist position.
- Angular divergence ($\sigma_{x', y'}$) is constant along the axis of propagation, as far as no optical elements are present.
- Emittance ($\varepsilon_x, \varepsilon_y$) is defined as $\sigma_{x, y} \times \sigma_{x', y'}$, which is equal to the area of the phase ellipse divided by π .

Fundamentals of Light and SR

- Why we need SR?
- Physical Quantity of Light
- Uncertainty of Light
- SR: Light from a Moving Electron

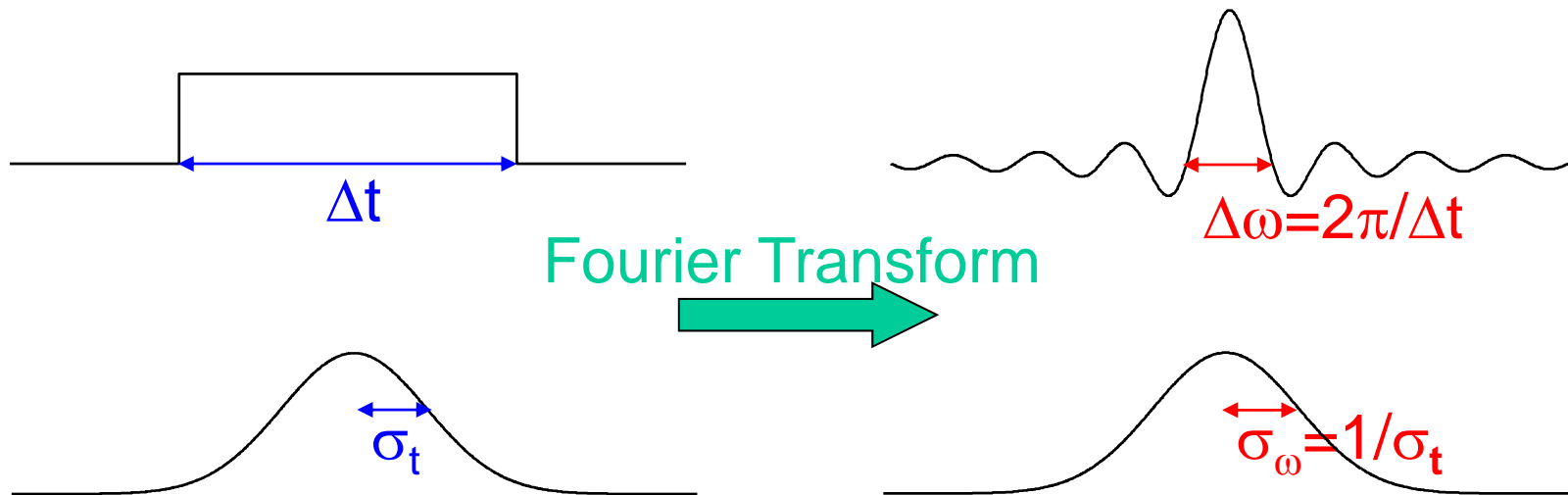
Uncertainty of Light

- The photon distribution in the 6D phase space $(x, y, x', y', t, \omega)$ gives us the full information on the properties of SR.
- Due to wave nature of light, however, we have two uncertainty relations to take care, which are well characterized by the Fourier transform.
- These relations impose two restrictions on SR, Fourier and Diffraction limits.

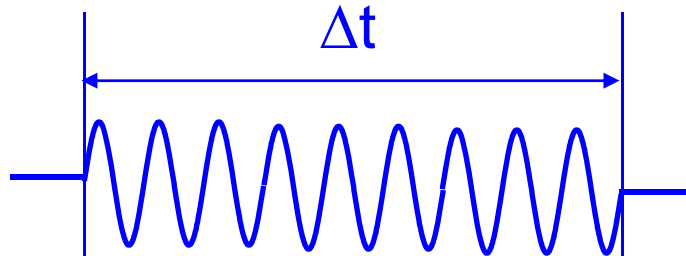
Fourier Transform: Example

Important Fourier Transform in SR Formulae

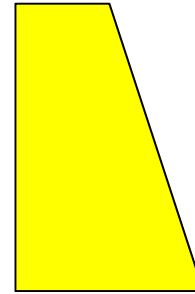
$F(t)$	$f(\omega) = \int_{-\infty}^{\infty} F(t) e^{i\omega t} dt$
$\begin{cases} 1/\Delta t; & -\Delta t/2 \leq t \leq \Delta t/2 \\ 0; & t < -\Delta t/2, \Delta t/2 < t \end{cases}$	$\frac{\sin \omega \Delta t/2}{\omega \Delta t/2} \equiv \text{sinc}(\omega \Delta t/2)$
$\frac{1}{\sqrt{2\pi}\sigma_t} \exp\left(-\frac{t^2}{2\sigma_t^2}\right)$	$\exp\left(-\frac{\omega^2 \sigma_t^2}{2}\right)$



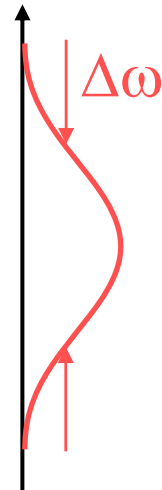
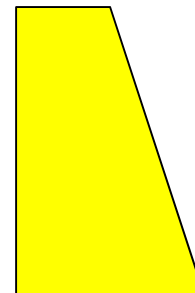
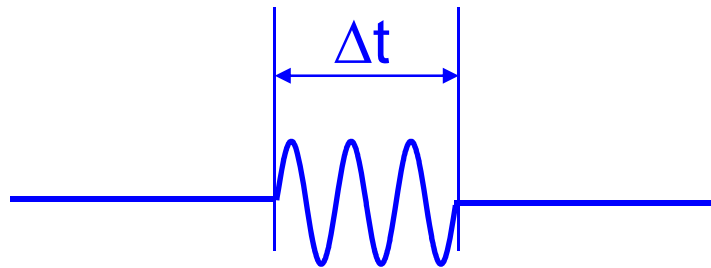
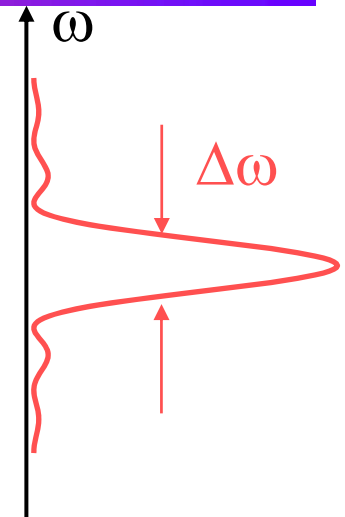
Spectrum of Light



Light with a finite
time duration of Δt



spectrometer =
temporal Fourier transform



If the injected light is
monochromatic, $\Delta\omega\Delta t = \text{const.}$

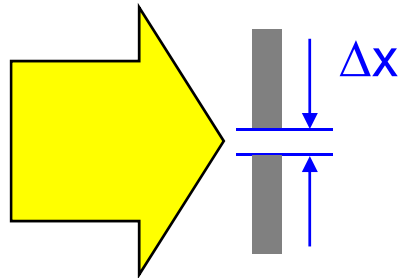
Fourier Limit of Light

- Temporal Fourier transform imposes

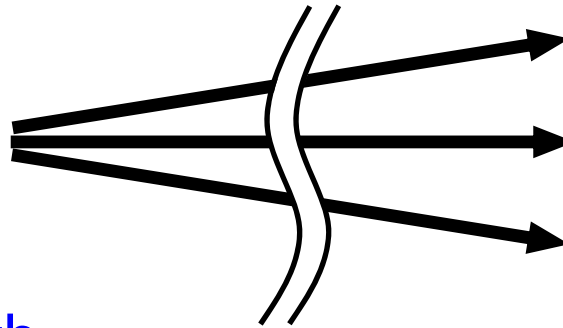
$$\Delta\omega \Delta t \geq \text{const.}$$

- Uncertainty of light in the (ω, t) plane.
- When equality holds, light is said to be
 - Fourier-limited
 - Temporally coherent
- Important to understand the spectral properties of SR.

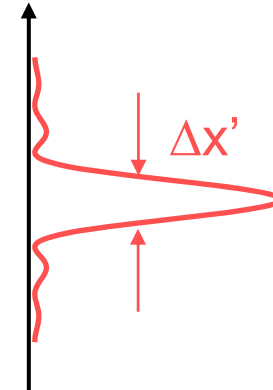
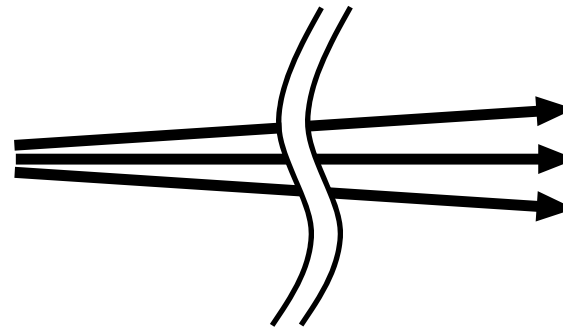
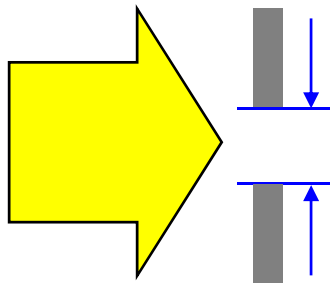
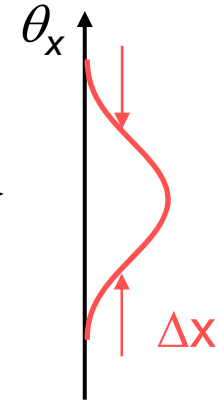
Diffraction Pattern of Light



Clipping by a slit with
a finite width of Δx



diffraction pattern in the far region
= spatial Fourier transform



If the injected light is a plane
wave, then $\Delta x \Delta x' = \text{const.}$

Diffraction Limit of Light

- Spatial Fourier transform imposes

$$\Delta x \Delta k_x \geq \text{const.}$$



$$\Delta x \Delta x' \geq \lambda \times \text{const.}$$

- Uncertainty of light in (x,x') plane
- When equality holds, light is said to be
 - Diffraction limited
 - Spatially coherent
- In the case of Gaussian beam,

$$\sigma_x \sigma_{x'} \geq \boxed{\lambda / (4\pi)} \text{ Natural emittance of light}$$

Fundamentals of Light and SR

- Why we need SR?
- Physical Quantity of Light
- Uncertainty of Light
- SR: Light from a Moving Electron

SR: Light from a Moving Electron

- Unlike the ordinary light source (sun, light bulb,...), the light emitter of SR (electron) is ultra-relativistic.
- The characteristics of SR is thus quite different because of relativistic effects.
- What we have to take care is:
 1. Speed-of-light limit
 2. Squeezing of light pulse
 3. Conversion of the emission angles

Speed-of-Light Limit

Within the framework of relativity, the velocity of an **electron** never exceeds the speed of light.

$$v/c = \beta = \sqrt{1 - \gamma^{-2}} \\ \sim 1 - \frac{1}{2\gamma^2}$$

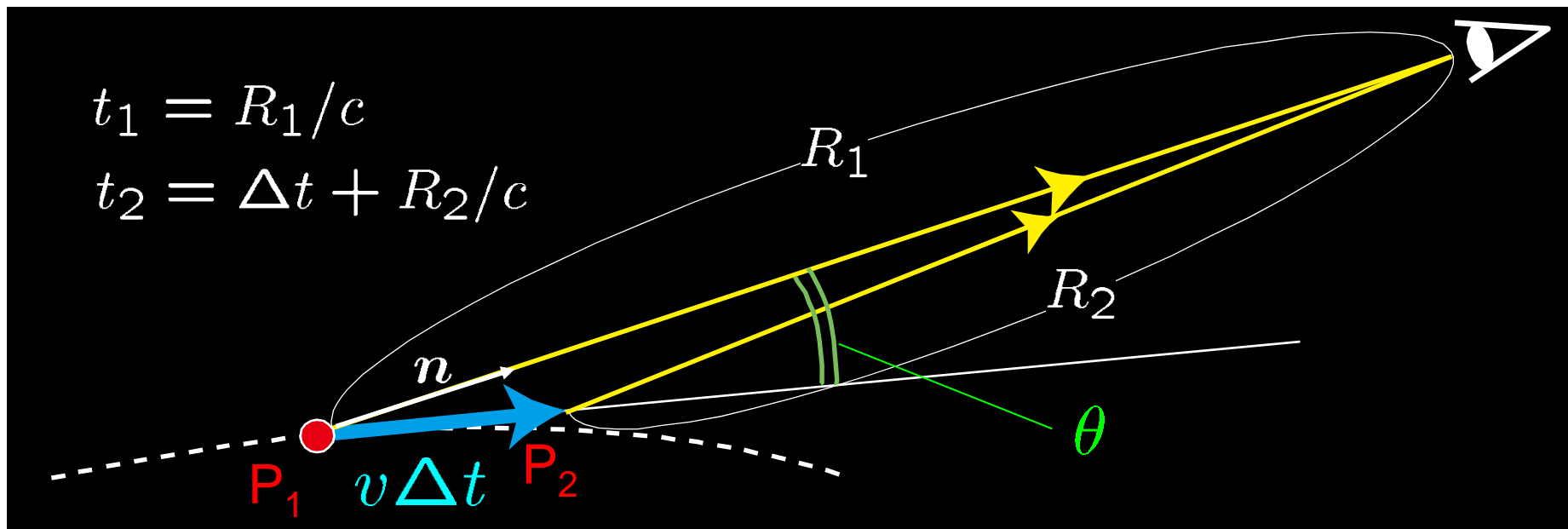
$$\gamma = \frac{E}{mc^2}$$

Energy	β
1MeV	0.941
10MeV	0.9988
100MeV	0.999987
8GeV	0.9999999998

:Lorentz Factor

(relative electron energy, $mc^2=0.511\text{MeV}$)

Squeezing of Light Pulse Duration



$$R_2 = \sqrt{(R_1)^2 + (v\Delta t)^2 - 2R_1v\Delta t \cos \theta}$$

$$\sim R_1 - (\mathbf{v} \cdot \mathbf{n})\Delta t$$

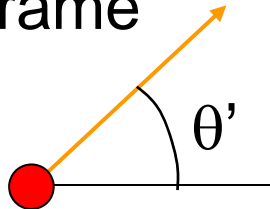
$$\Delta\tau = t_2 - t_1 = \Delta t + R_2/c - R_1/c$$

$$= \Delta t \boxed{(1 - \beta \cdot \mathbf{n})} = \boxed{\frac{\Delta t}{2\gamma^2}} \quad \gamma \gg 1, \theta = 0$$

time squeezing

Conversion of Emission Angles

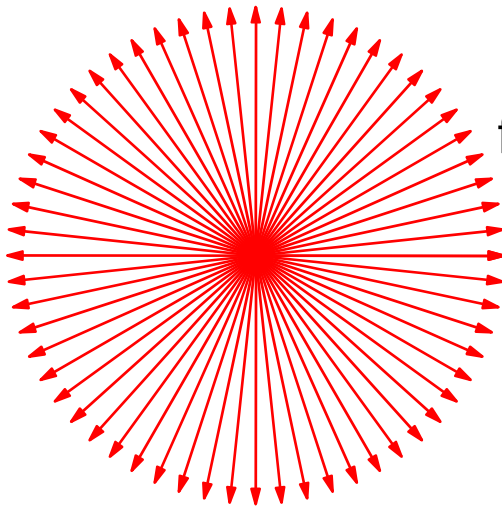
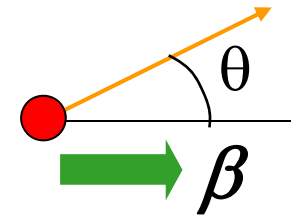
rest frame



how to convert?

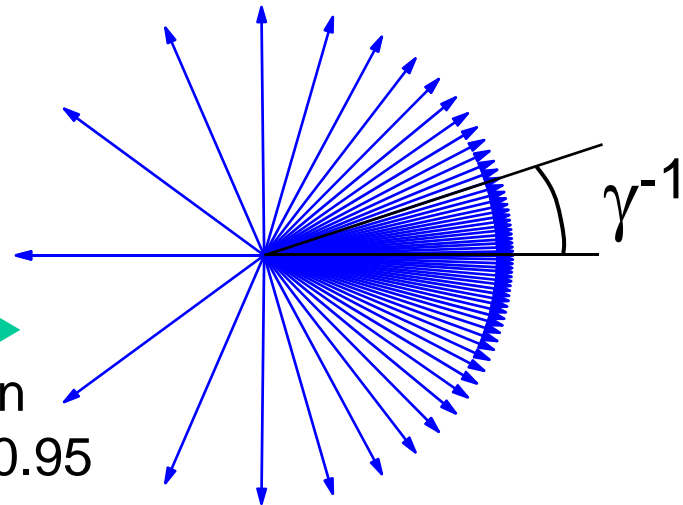
$$\theta = \tan^{-1} \left(\frac{\gamma^{-1} \sin \theta'}{\beta + \cos \theta'} \right)$$

lab. frame



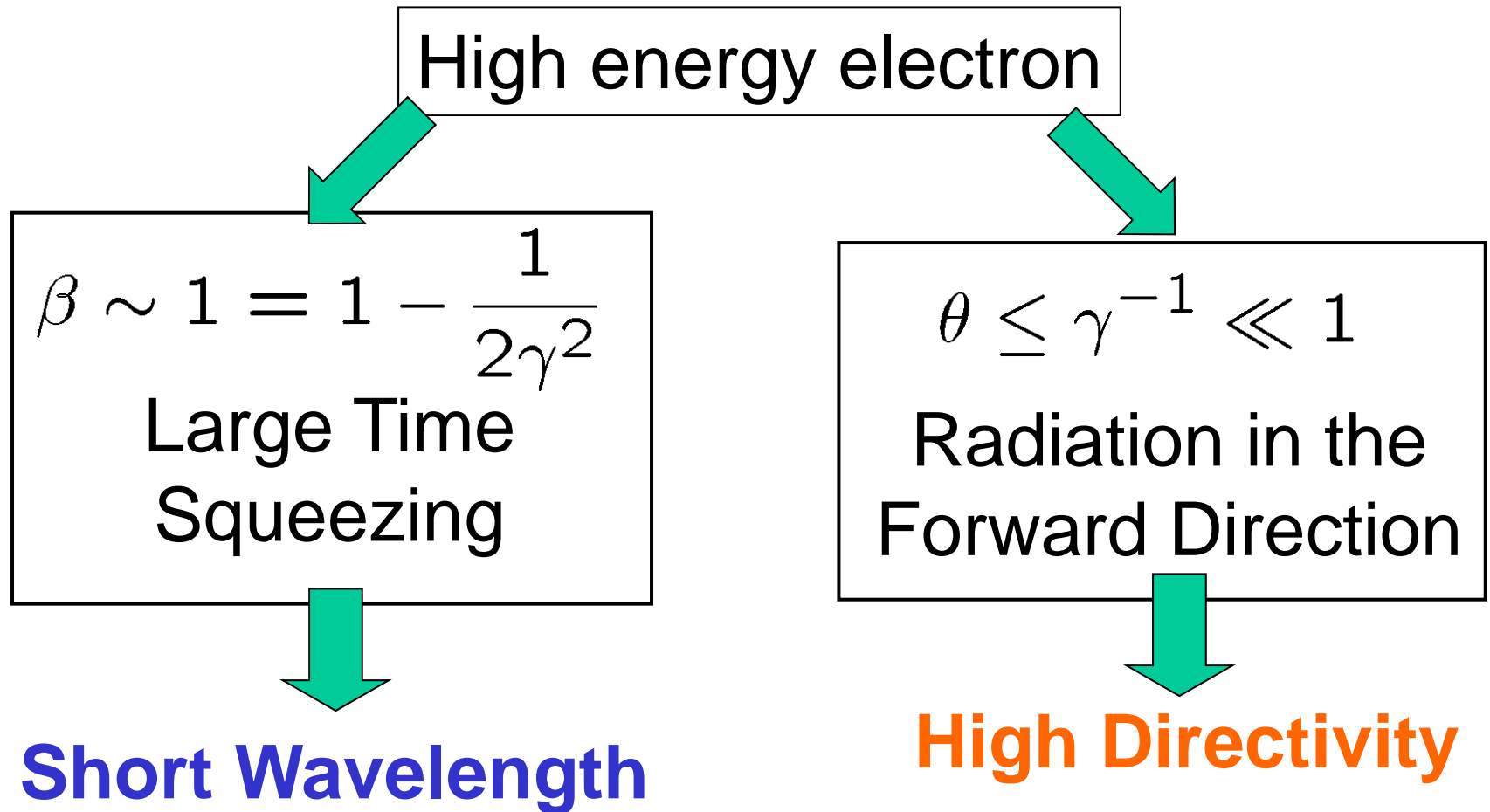
Isotropic emission
from a rest electron

Emission from an
electron with $\beta=0.95$



Light emitted from a moving object
($\beta \sim 1$) concentrates within γ^{-1}

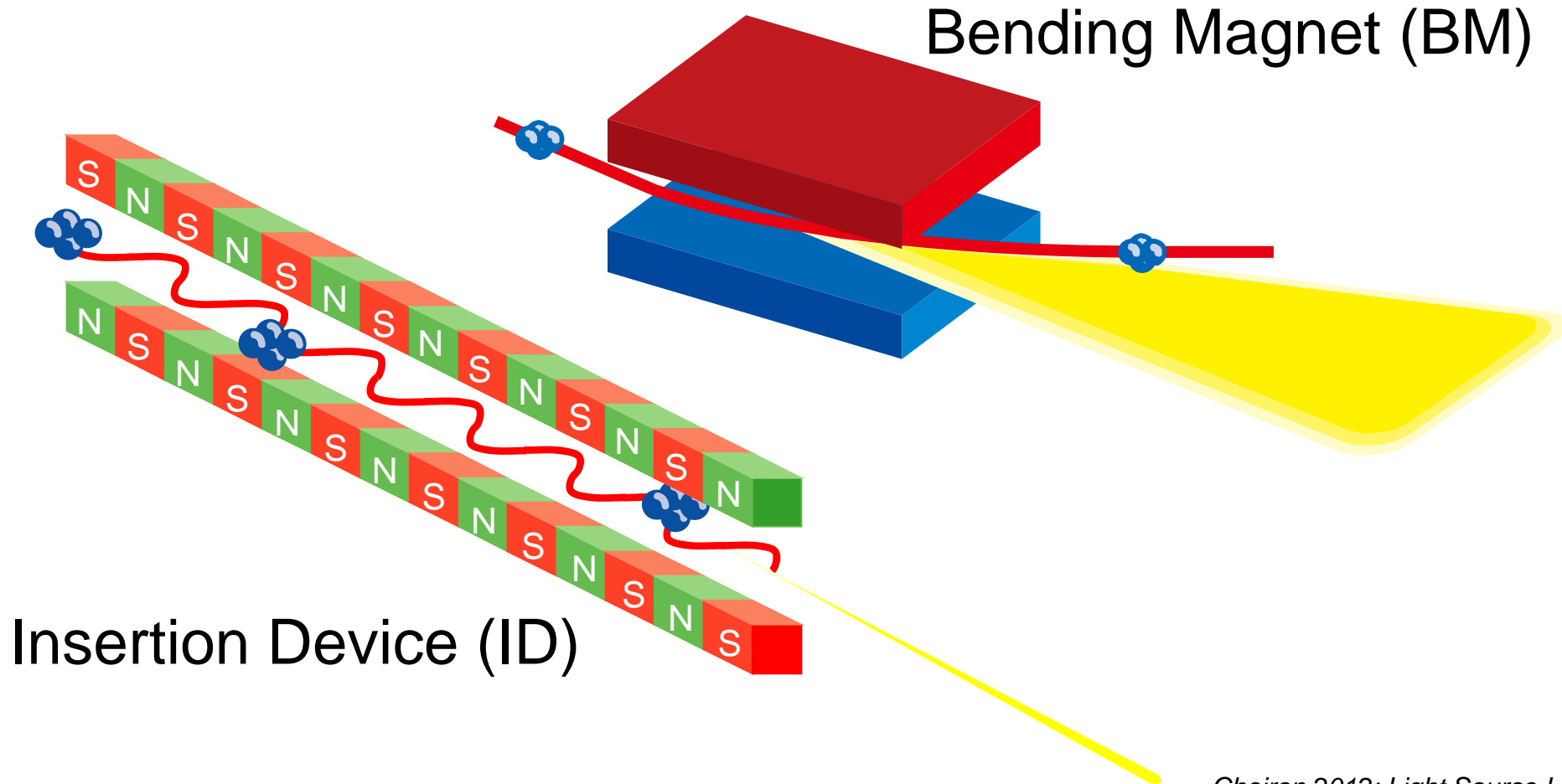
SR from a High-Energy Electron



Overview of SR Light Source

What is SR Light Source?

Magnets to deflect the electron beam and generate SR.



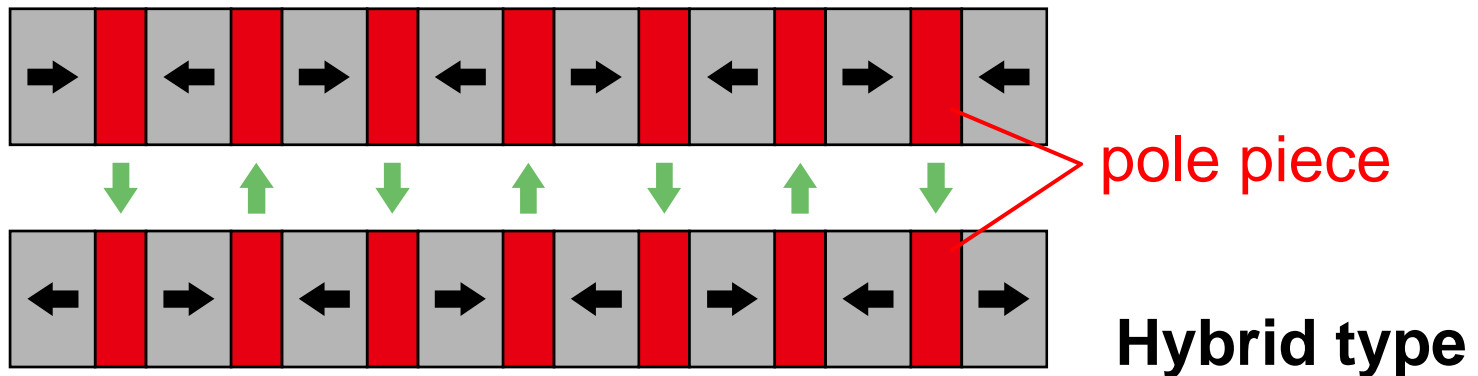
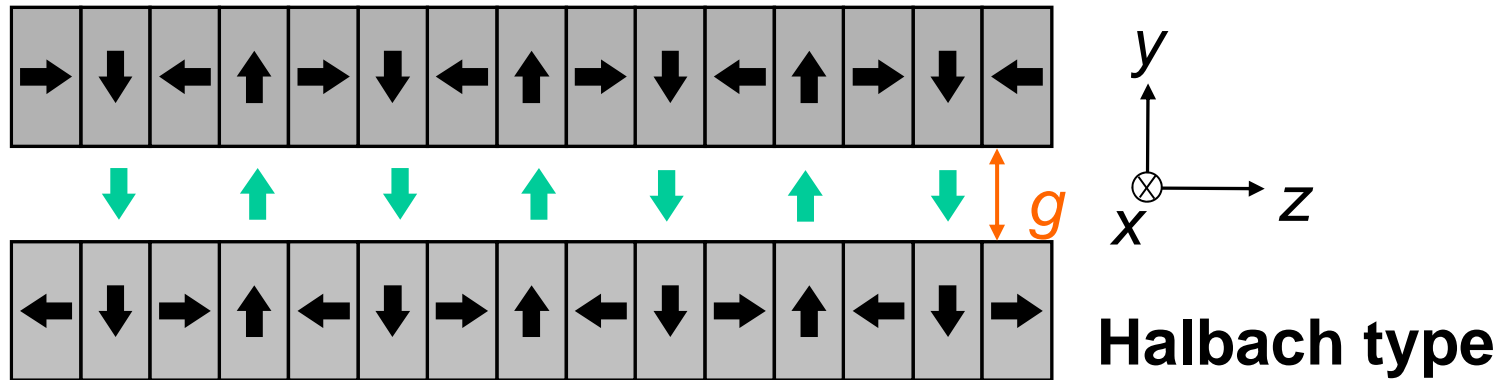
Bending Magnet

- One of the accelerator components in the storage ring.
- Generate uniform field to guide the electron beam into a circular orbit.
- EMs combined with highly-stable power supplies are adopted in most BMs due to stringent requirement on field quality and stability.
- Superconducting magnets are used in a few facilities in pursuit of harder x rays.

Insertion Device

- Installed (inserted) into the straight section of the storage ring between two adjacent BMs.
- Generate a periodic magnetic field to let the injected electron beam move along a periodic trajectory.
- Most IDs are composed of PMs, while EMs are used for special use such as helicity switching.
- Classified into **wigglers** and **undulators**.

Magnetic Circuit of IDs

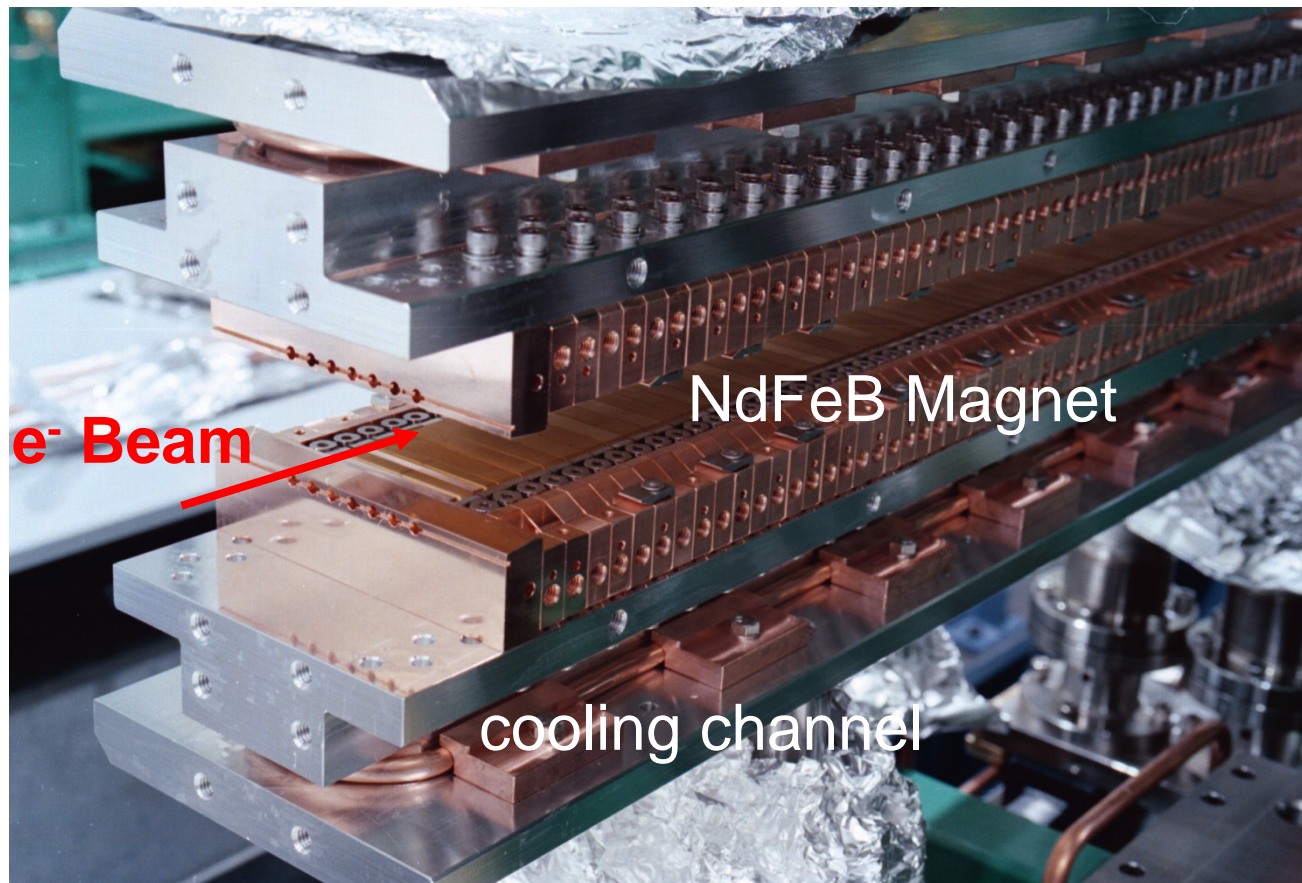


In each type, a sinusoidal magnetic field is obtained:

$$B_y(z) \sim B_0(B_r, g/\lambda_u) \sin\left(\frac{2\pi z}{\lambda_u}\right)$$

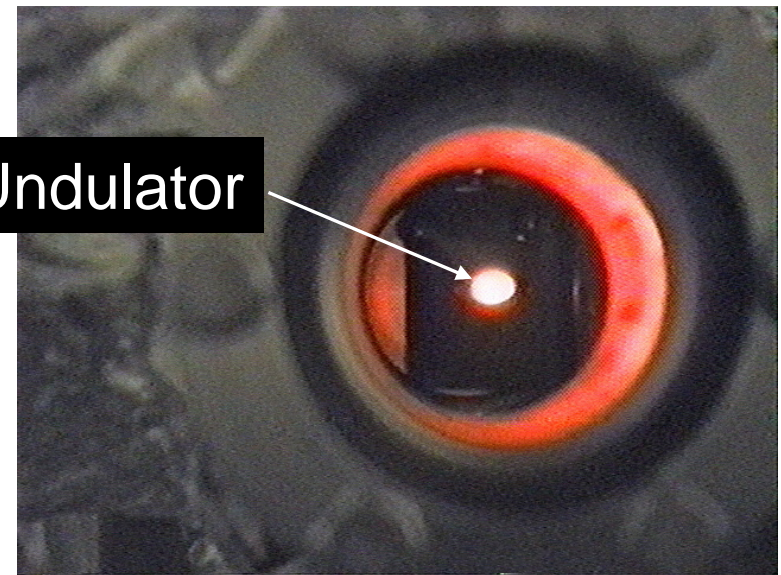
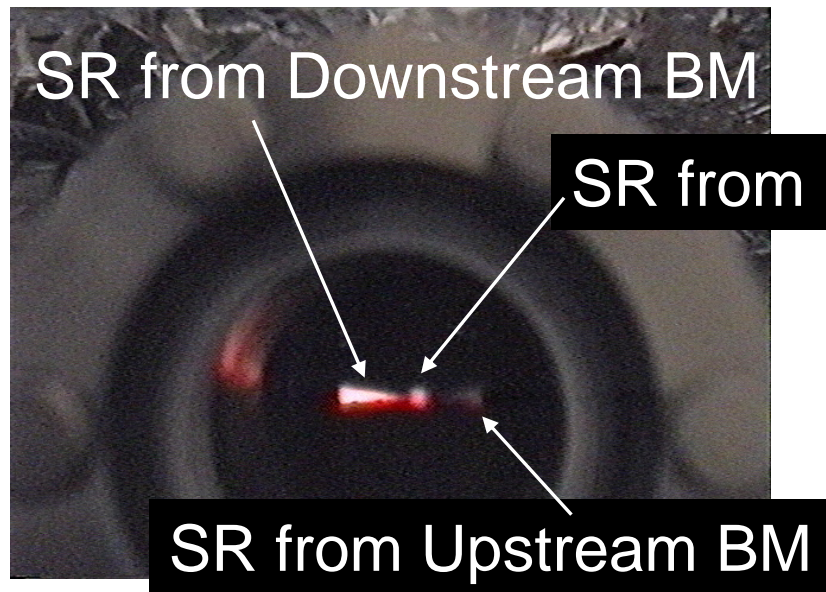
Example of ID Magnets

Halbach-type Magnet Array for SPring-8 Standard Undulators

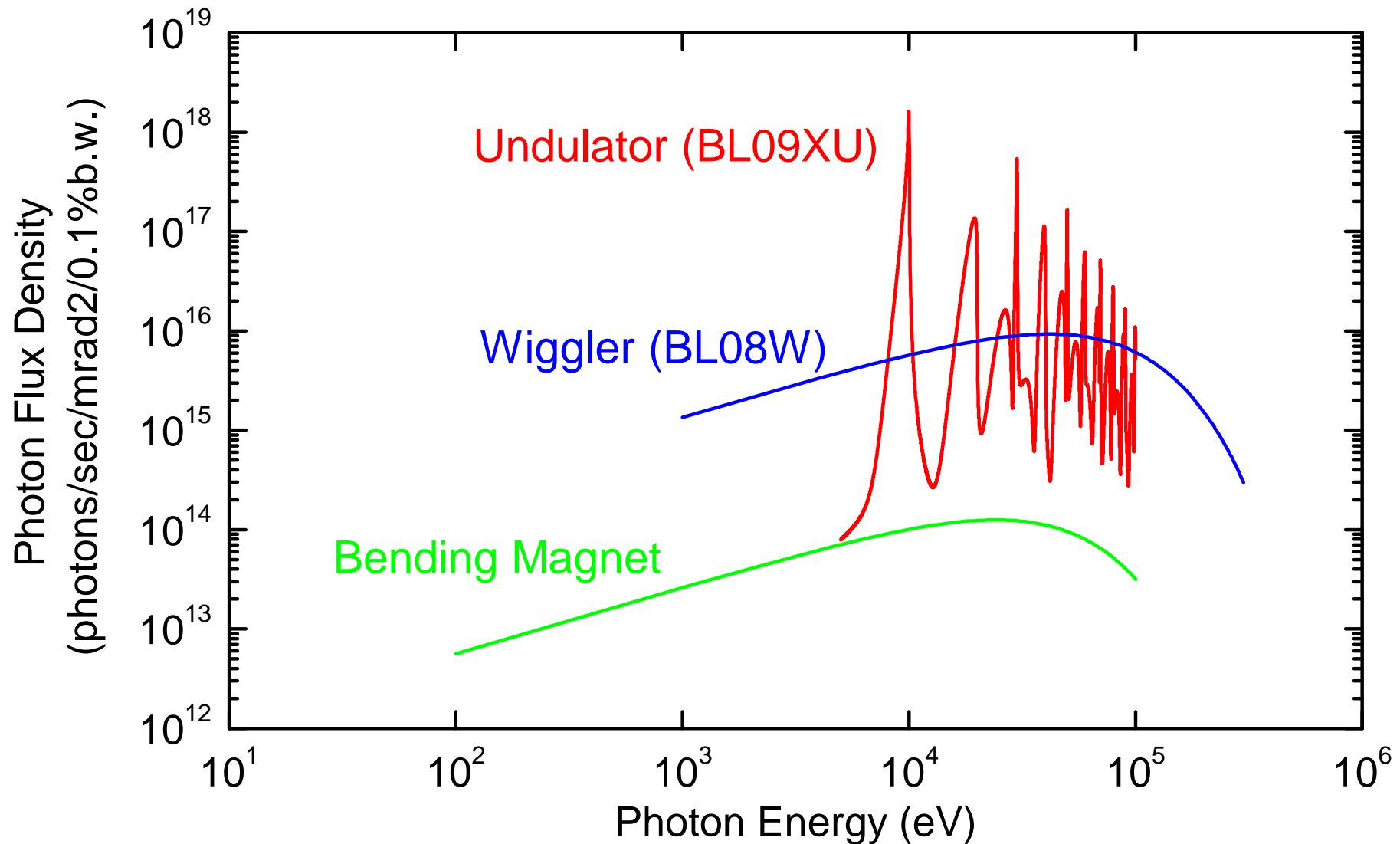


Example of SR Image

BL41XU@SP-8, first image of SR
with a fluorescent screen ($<0.1\text{mA}$)



Comparison of Light Sources

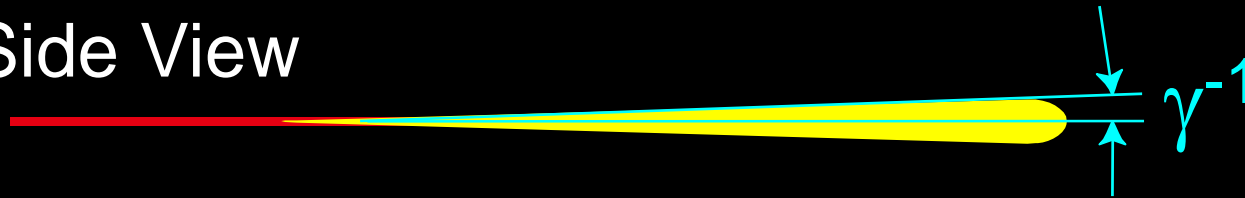


Characteristics of SR (1)

- Radiation from BMs

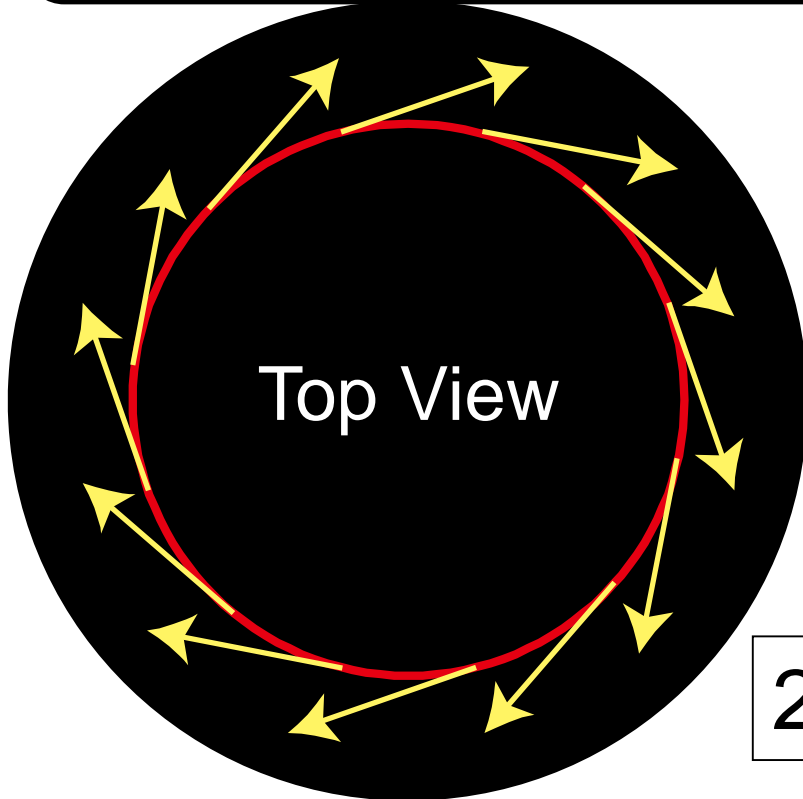
Directivity of BM Radiation

Side View



↑ High directivity in the vertical plane
($\sigma_y \sim \gamma^{-1} \sim 64 \mu\text{rad}$ @ SP-8)

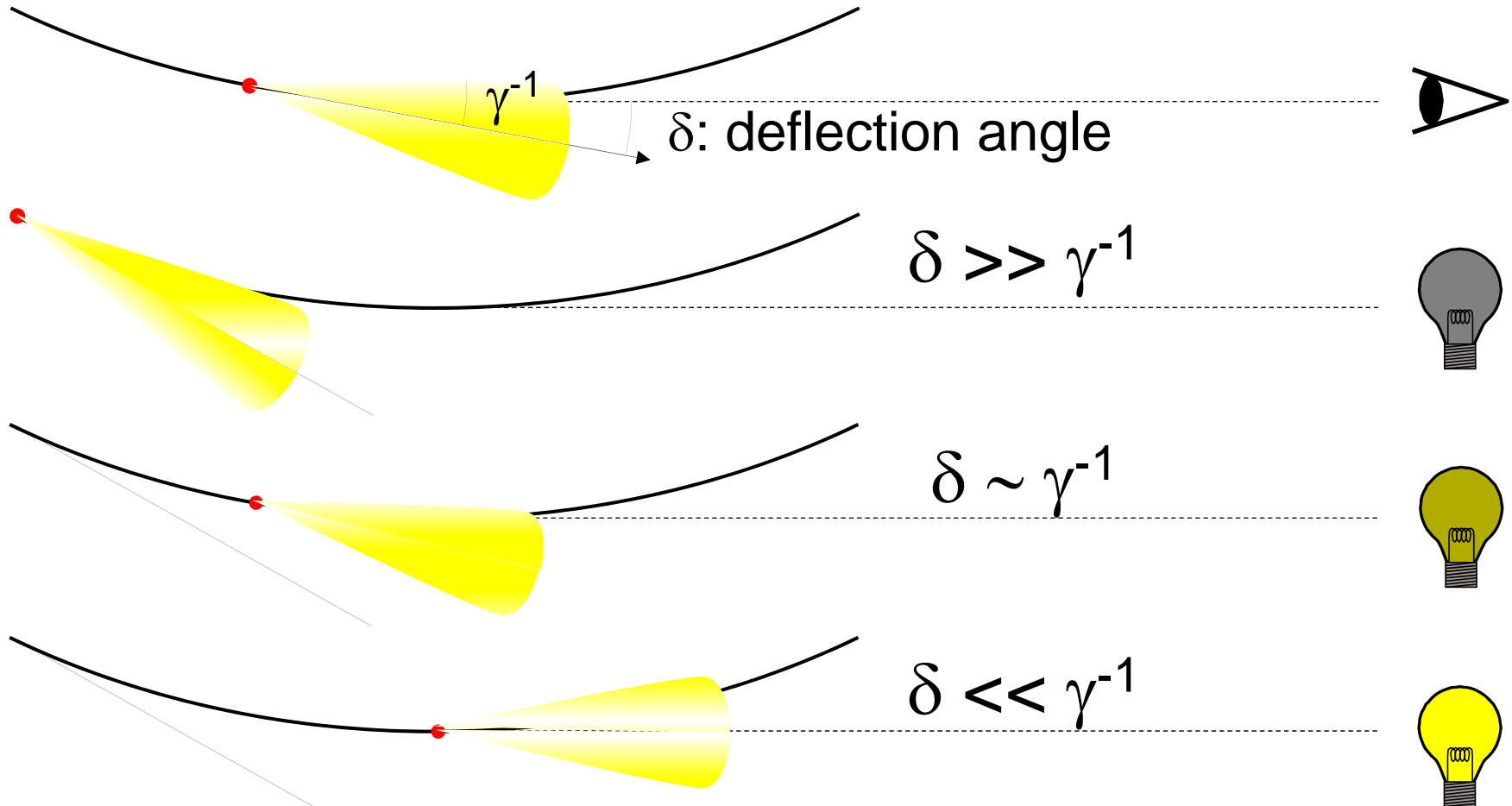
Top View



← Isotropic in the orbital plane

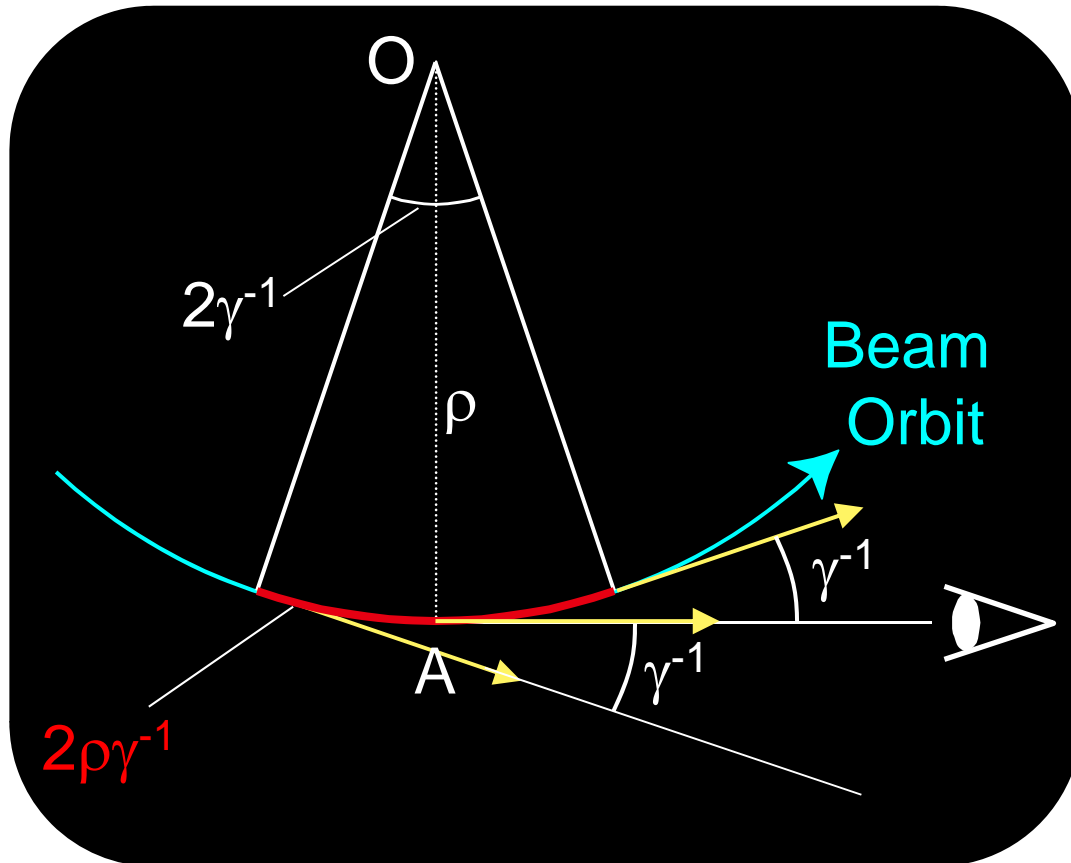
2-dimensional directivity

Spectrum of BM Radiation (1)



Photons emitted when $\delta < \gamma^{-1}$
are detected by the observer

Spectrum of BM Radiation (2)



Major contribution of radiation is from the portion painted red



Pulse duration for e^-
 $\Delta t = 2\rho\gamma^{-1}/c$

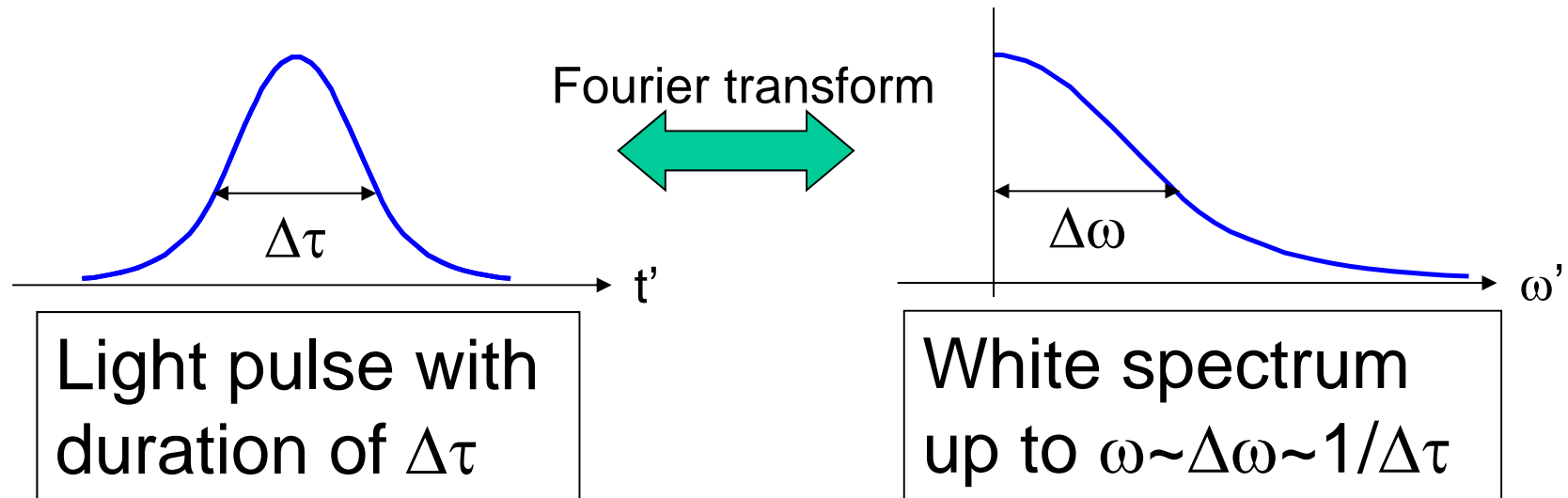


squeezing

Pulse duration for observer

$$\Delta\tau = \frac{\Delta t}{2\gamma^2} = \frac{\rho}{\gamma^3 c}$$

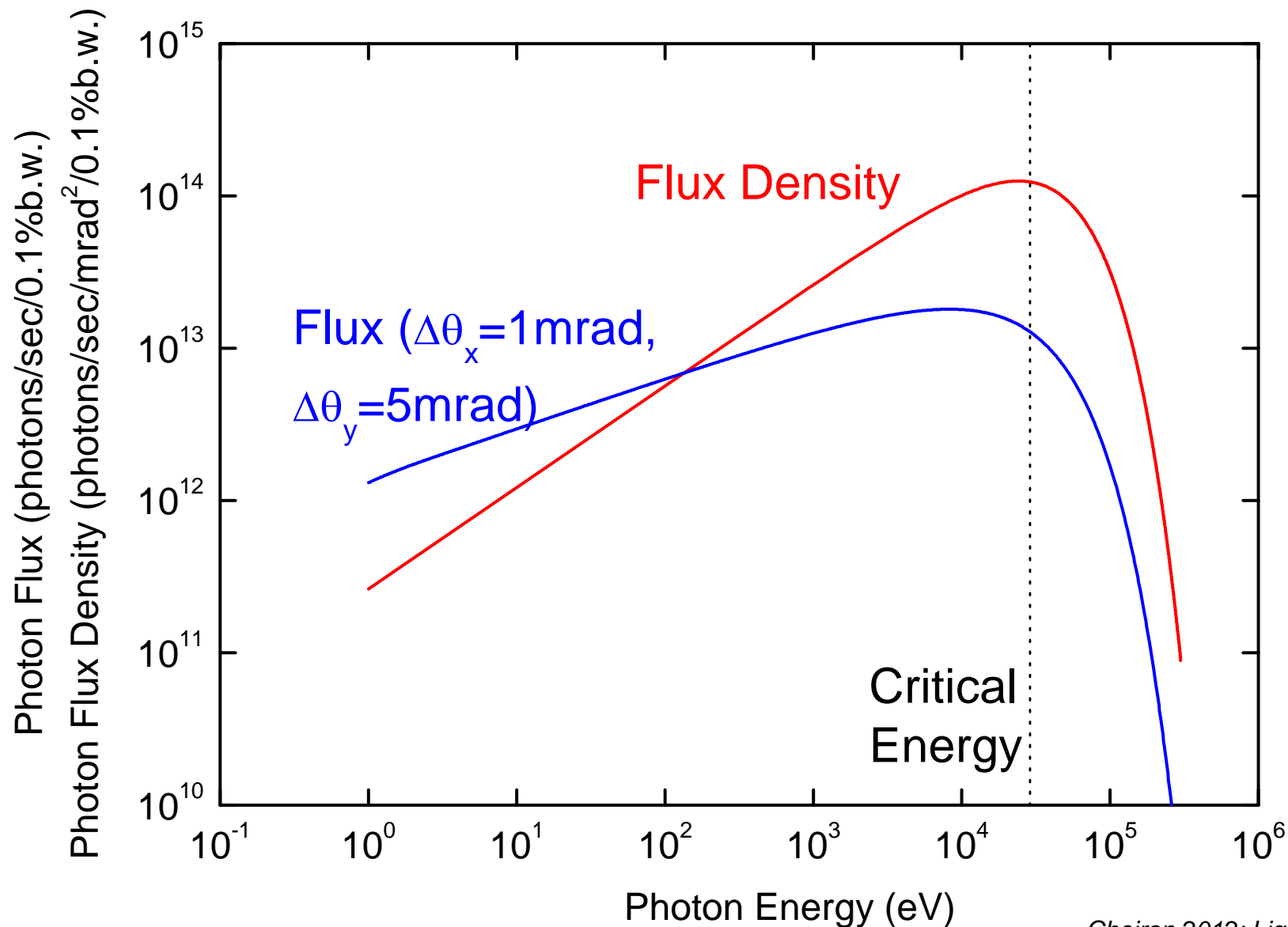
Spectrum of BM Radiation (3)



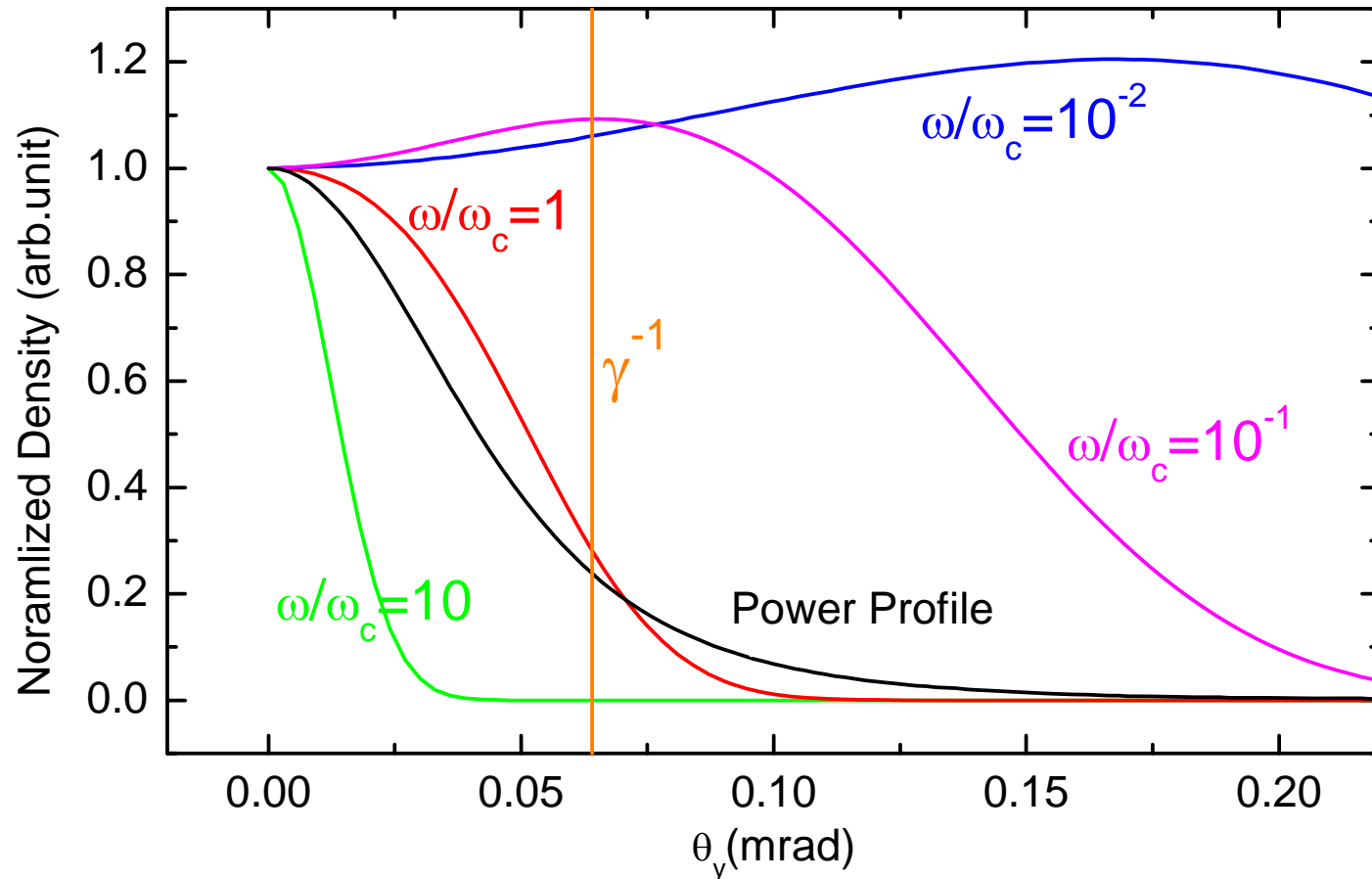
- By definition, $\omega_c = (3/2)/\Delta\tau = 3\gamma^3 c / 2\rho$ is called “critical frequency” of SR, which gives a criterion of the maximum energy of SR from a BM.
- In practical units,

$$\hbar\omega_c(\text{keV}) = 0.665 E_e^2(\text{GeV}) B(\text{T})$$

Example of Spectrum



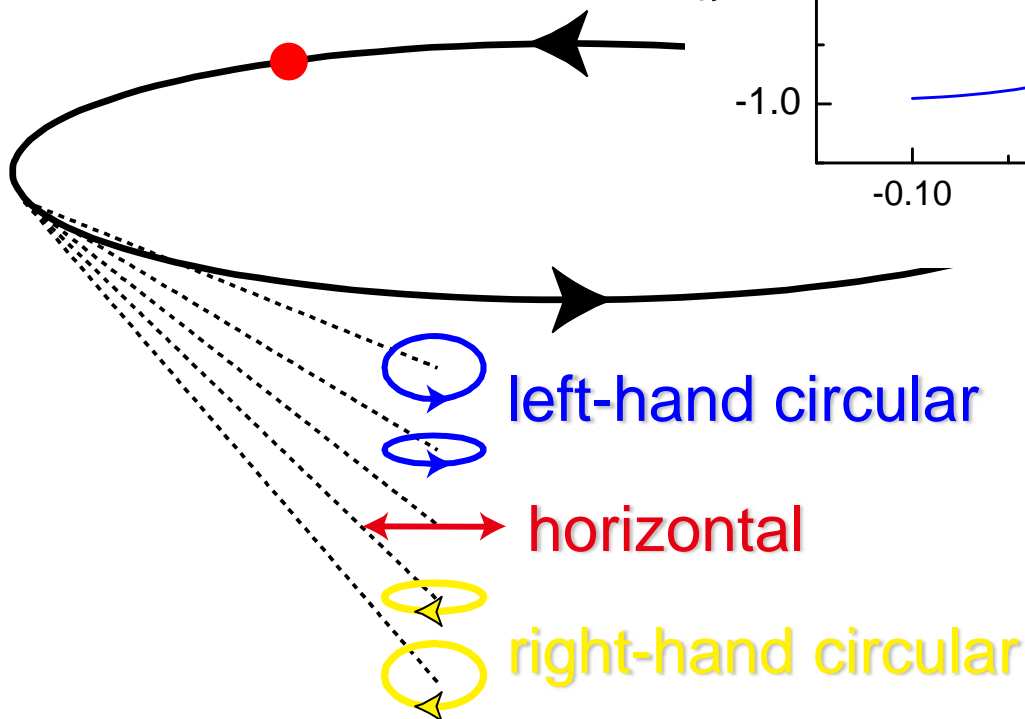
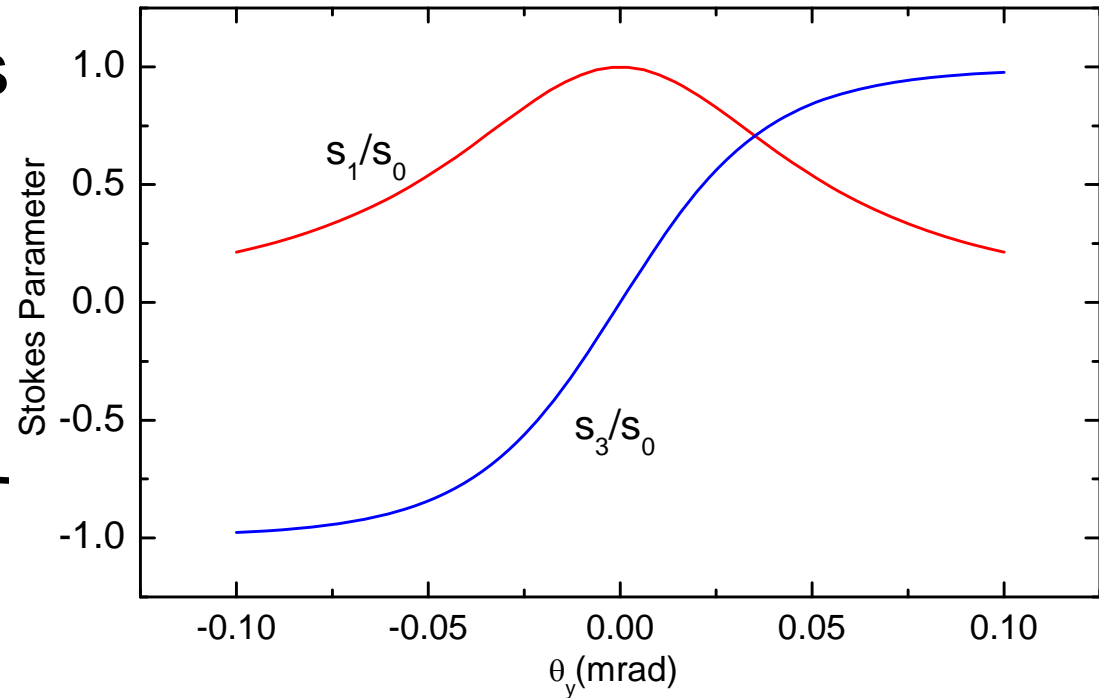
Angular Profile of BM Radiation



- power profile \sim flux profile @ $\omega/\omega_c=1$
- larger angular divergence for lower energy

Polarization of BM Radiation

Stokes parameters
of BM radiation
along vertical axis



Polarization state
reflects the apparent
motion of electron.